Chapter 9

Right Triangles and Trigonometry

Section 5
Trigonometric Ratios

GOAL 1: Finding Trigonometric Ratios

A **trigonometric ratio** is a ratio of the lengths of two sides of a right triangle. The word *trigonometry* is derived from the ancient Greek language and means measurement of triangles. The three basic trigonometric ratios are **sine, cosine,** and **tangent,** which are abbreviated as *sin*, *cos*, and *tan*, respectively.

SOH CAH TOA

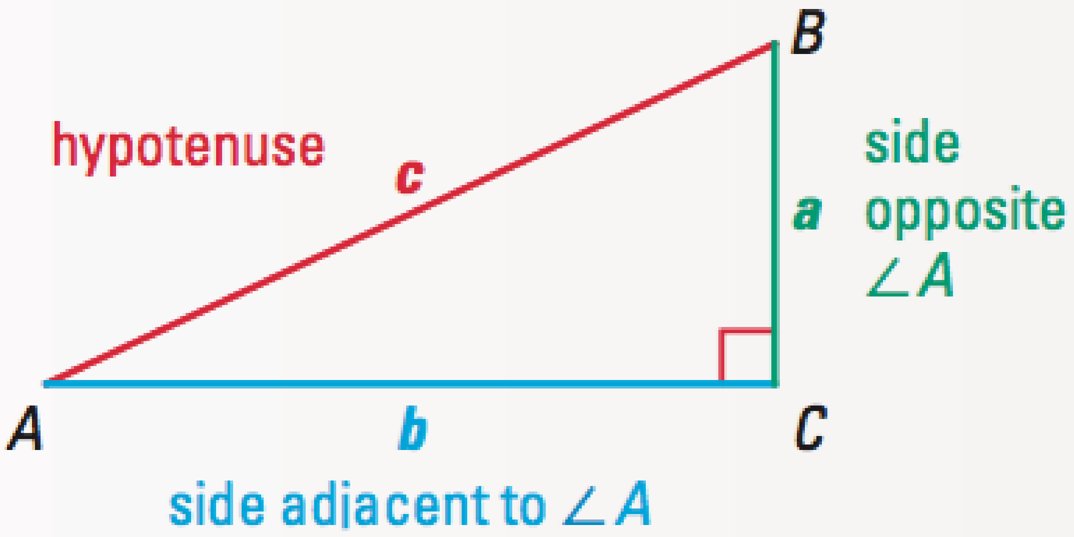
TRIGONOMETRIC RATIOS

Let $\triangle ABC$ be a right triangle. The sine, the cosine, and the tangent of the acute angle $\angle A$ are defined as follows.

$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$



Example 1: Finding Trigonometric Ratios

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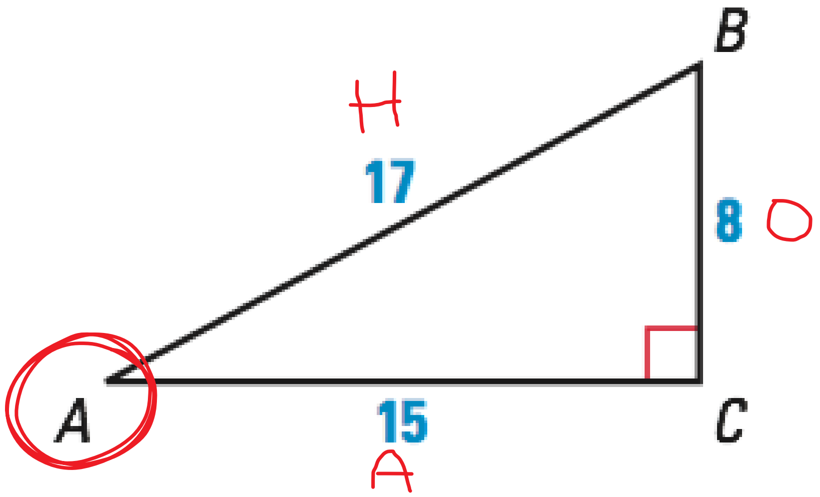
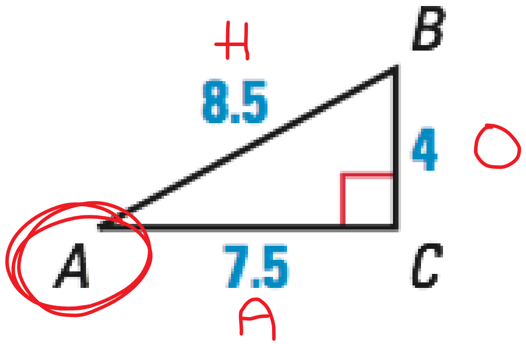
Compare the sine, the cosine, and the tangent ratios for <A in each triangle below.





sin = 8/17

cos = 15/17

tan = 8/15

sin = 4/8.5

cos = 7.5/8.5

tan = 4/7.5

they are the same

Example 2: Finding Trigonometric Ratios

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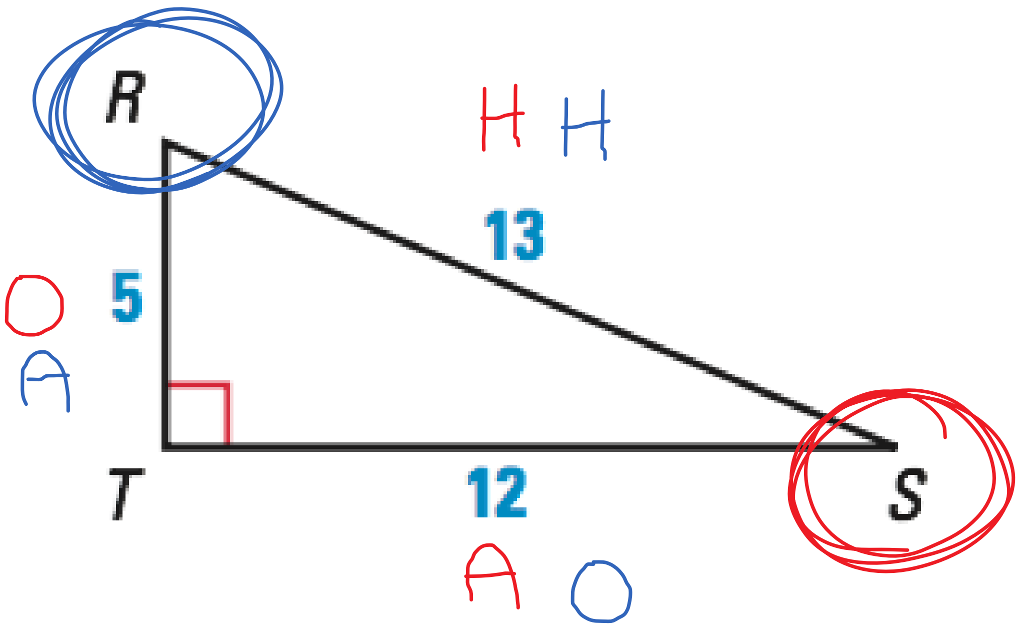
Find the sine, the cosine, and the tangent of the indicated angle.

a) <S

sin = 5/13

cos = 12/13

tan = 5/12



b) <R

sin = 12/13

cos = 5/13

tan = 12/5

Notes:

Trigonometric ratios are frequently expressed as decimal approximations.

You can find trigonometric ratios for 30*, 45*, and 60* by applying what you know about special right triangles.

Example 3: Trigonometric Ratios for 45*

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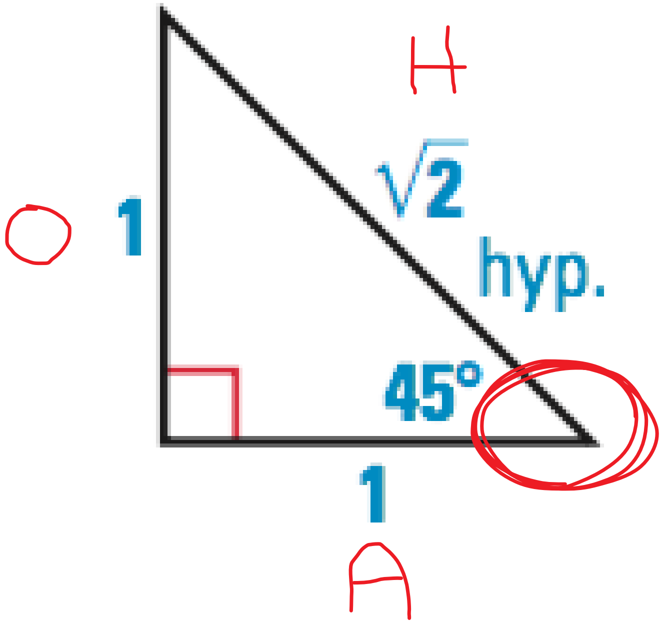
Find the sine, the cosine, and the tangent of 45*.

$$\sin \rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$$

$$\cos \rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \frac{\sqrt{2}}{2}$$

$$\tan \rightarrow \frac{1}{1} \rightarrow 1$$



Example 4: Trigonometric Ratios for 30*

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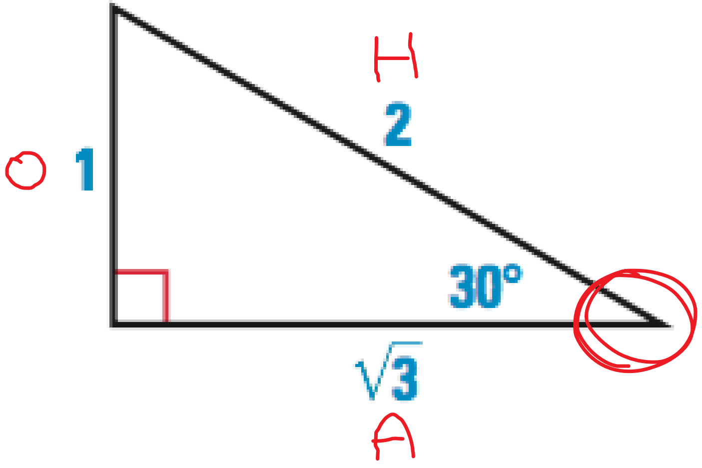
Find the sine, the cosine, and the tangent of 30*.



$\sin \rightarrow \frac{1}{2}$

$\cos \rightarrow \frac{\sqrt{3}}{2}$

$\tan \rightarrow \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{\sqrt{3}}{3}$

Example 5: Using a Calculator

You can use a calculator to approximate the sine, the cosine, and the tangent of 74°. Make sure your calculator is *in degree mode*. The table shows some sample keystroke sequences accepted by most calculators.

Sample keystroke sequences	Sample calculator display	Rounded approximation
74 SIN or SIN 74 ENTER	0.961261695	0.9613
74 COS or COS 74 ENTER	0.275637355	0.2756
74 TAN or TAN 74 ENTER	3.487414444	3.4874

phone

calc

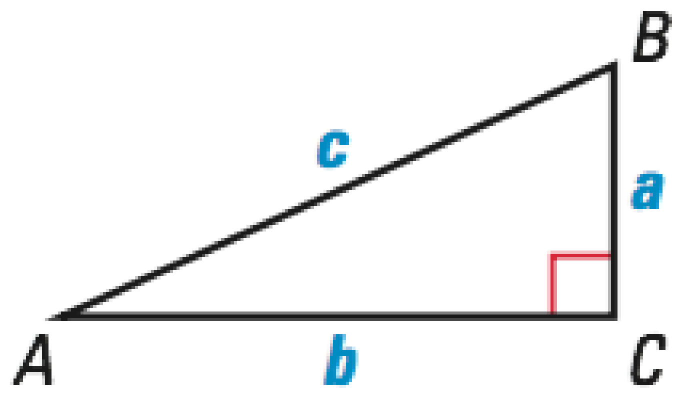
If you look back at Examples 1–5, you will notice that the sine or the cosine of an acute angle is always less than 1. The reason is that these trigonometric ratios involve the ratio of a leg of a right triangle to the hypotenuse. The length of a leg of a right triangle is always less than the length of its hypotenuse, so the ratio of these lengths is always less than one.

Because the tangent of an acute angle involves the ratio of one leg to another leg, the tangent of an angle can be less than 1, equal to 1, or greater than 1.

TRIGONOMETRIC IDENTITIES A trigonometric identity is an equation involving trigonometric ratios that is true for all acute angles. You are asked to prove the following identities in Exercises 47 and 52:

$$(\sin A)^2 + (\cos A)^2 = 1$$

$$\tan A = \frac{\sin A}{\cos A}$$



GOAL 2: Using Trigonometric Ratios in Real Life

Suppose you stand and look up at a point in the distance, such as the top of the tree in Example 6. The angle that your line of sight makes with a line drawn horizontally is called the **angle of elevation.**

Example 6: Indirect Measurement

FORESTRY You are measuring the height of a Sitka spruce tree in Alaska. You stand 45 feet from the base of the tree. You measure the angle of elevation from a point on the ground to the top of the tree to be 59°. To estimate the height of the tree, you can write a trigonometric ratio that involves the height *h* and the known length of 45 feet.

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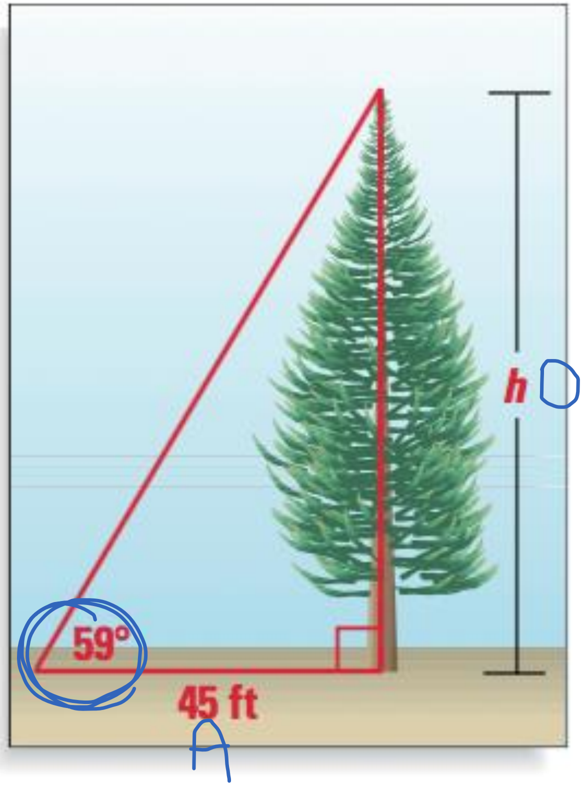
TOA



$$45 \cdot \tan 59 = \frac{h}{\cancel{45}} \times 45$$

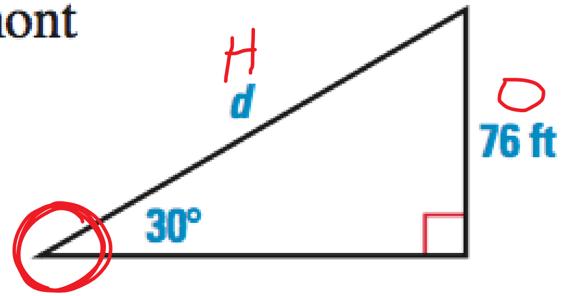
$$45(\tan 59) = h$$

$$74.89 = h$$

(feet)

Example 7: Estimating a Distance

ESCALATORS The escalator at the Wilshire/Vermont Metro Rail Station in Los Angeles rises 76 feet at a 30° angle. To find the distance *d* a person travels on the escalator stairs, you can write a trigonometric ratio that involves the hypotenuse and the known leg length of 76 feet.



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TOA

$$\sin 30 = \frac{76}{d}$$

$$d = \frac{76}{\sin 30}$$

$$d = 152 \text{ ft}$$

*variable is on top → multiply

*variable in on bottom → divide

EXIT SLIP